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Internal stress and unloading experiments in creep

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Internal stresses are developed during deformation and have an important role in determining the mechanical properties and, in particular, the creep properties of crystalline materials. The strain transient dip test is the generally accepted method for the determination of internal stresses developed during creep. The strain transient dip test has been analysed using a number of very general creep models and it is concluded that, for glide-controlled creep, the dip test can only be interpreted if the relation between dislocation velocity and the force on the dislocation is linear. When this is the case it measures not an average internal stress but an average back stress for all the dislocations, mobile and immobile, where the back stress is the resolved component of the internal stress plus the glide component of the line tension force divided by the Burgers vector. The dip test does not allow separation of the back stress into internal stress and line tension components. For recovery models the results of the dip test cannot be simply interpreted because expressions for the creep rate do not define a unique average internal stress or back stress. However, for the recovery model in which strain occurs by athermal or jerky glide there will be a reverse yield stress, i.e. there will be a stress reduction below which there will be "instantaneous" reverse strain followed by reverse creep. By averaging the instability condition for all the dislocations participating in jerky glide it is shown, subject to assumptions, that the sum of the average internal stress experienced by dislocations involved in both forward and reverse creep can be obtained from the reverse yield stress. Separate values for these internal stresses cannot be obtained, however. Determination of the reverse yield stress for recovery creep is the experiment equivalent to the strain transient dip test for glide-controlled creep.

1. Introduction

It has long been recognized that internal stresses are developed during the deformation of materials and that these internal stresses have an important role in determining mechanical properties [1–6]. At any point in the material after deformation there is a local effective stress which is the sum of the stress due to the externally applied forces and the internal stress. The mechanics and kinetics of the deformation and recovery processes occurring in this local region are determined by, amongst other things, the local effective stress. The mechanical behaviour of the material body as a whole is some sort of total or average of the behaviour of all its local regions. Hence, after the passage from the microscopic regions to the macroscopic material body it may still be possible to discern an effective stress which is different from the applied stress. At this macroscopic level the applied (tensile) stress is σ_a and the average internal stress (also regarded as tensile) is $-\sigma_i$, where the convention of a negative sign is used to make the internal stress, which is opposite in sign to the applied stress, into a positive quantity. The macroscopic effective (tensile) stress, σ_e (sometimes given the symbol σ^*), is thus given by

$$\sigma_e = \sigma_a - \sigma_i \quad (1)$$

Internal stresses arise because plastic deformation is heterogeneous even in single-phase materials. In such materials deformation often develops walls of densely tangled dislocations outlining cells which are relatively free of dislocations. The cell interiors are "soft" and easily deformed while the high dislocation density walls are "hard" and resist plastic deformation. When such a structure is stressed the soft regions deform plastically while the hard regions can only deform elastically. Geometrically necessary dislocations [7] accumulate on the cell walls as a result of the gradient of plastic strain between walls and interior. This has been discussed in detail by Mughrabi [8]. The geometrically necessary dislocations lead to internal stresses in the same direction as the applied stress in the cell walls and internal stresses opposing the applied stress in the cell interiors. The view that internal stresses arising from cell walls play an important part in creep has been put forward by a number of workers [3, 4].

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Even more pronounced effects are possible in multiphase systems if the phases have markedly different plastic properties. The role of internal stresses in the work-hardening of alloys hardened by dispersions of small non-deformable particles is fairly well understood [9]. The geometrically necessary dislocations giving rise to the internal stresses are, in this case, Orowan loops around the particles. These ideas have been applied to the creep of nickel-based alloys containing γ' precipitates [5, 6] and aluminium lithium-based alloys containing δ', S and T_1 precipitates [10, 11]. The role of internal stresses in the creep of composite materials has also been studied [12, 13].

One of the effects of an internal stress is an unexpectedly high dependence of the creep rate on stress. It was pointed out by Weertman [14] that the creep rate should be proportional to σ_a^3 for a uniform distribution of dislocations, i.e. one for which internal stresses are minimal. Stress exponents of around 5 are typical for Class II single-phase alloys and can be very much larger for multi-phase alloys. The difference can be attributed to the fact that there is an internal stress which is constant, or changes relatively slowly, so that changes in the applied stress give much larger fractional changes in effective stress with correspondingly large changes in creep rate.

Measurements of the macroscopic internal stress during creep have been carried out using what are usually called "dip tests" [5, 6, 10, 11, 15–19], the most important being the strain transient dip test. This latter technique involves making a series of stress reductions of progressively increasing size during a creep test and observing the resulting creep rates following the reductions. Small stress reductions give a reduced positive creep rate but after a sufficiently large reduction in stress, a negative creep rate may result temporarily. This is usually taken to indicate that the effective stress has become negative. There is a stress reduction which will give zero creep rate, and it is assumed that this implies that the new applied stress is equal to the internal stress in accordance with Equation 1. This stress reduction is usually obtained by interpolation. The strain transient dip test has been modified by other workers [19], but the experimental values of σ_i are only slightly affected by this [20].

Discussion of these phenomena is complicated by the lack of carefully defined and universally agreed terms. Thus different authors use the terms friction stress, threshold stress, internal stress, back stress and effective stress in different and inconsistent ways. One of the aims of the present paper is to give a consistent and hopefully complete set of descriptions and definitions which can be used to give a full discussion of the phenomena.

The main aim of this paper, however, concerns the meaning of the values obtained from the strain transient dip test for the two broad classes of creep model – glide control and recovery control. Because, by definition, internal stresses exist in a body without external forces on its surface, the volume average of each component of internal stress over the whole body must be zero. The average internal stress experienced by the moving dislocations must therefore be some kind of weighted average. Furthermore the strain transient dip test involves observations of the strain rate and hence it is to be expected that the relationship between effective stress and the average area per unit time swept by each element of dislocation line will also be involved in defining the average effective stress. It has never been formally demonstrated that the value of the effective stress determined by the strain transient dip test is, in fact, the average value which when inserted in the correct expression for the creep rate in terms of the effective stress (if this were known) would give the correct creep rate.

2. Basic definitions

For simple loading systems the applied stress is constant throughout the volume of the creep specimen, at least until cavities and cracks arise. In constant stress creep tests it should also be independent of time. Internal stresses, often called residual stresses by engineers when they are on a macroscopic scale, are zero, or at least small, initially. They increase with time and vary from point to point in the specimen as the dislocation distribution becomes more and more heterogeneous.

The force on a dislocation is defined as minus the rate of change of the thermodynamic potential with dislocation displacement. Under conditions of constant temperature and constant external surface forces the appropriate thermodynamic potential will be the generalized Gibbs' free energy. The force can be separated into a number of additive components arising from different contributors to the thermodynamic potential. The rate of change of the potential energy of the loading system gives rise to a force whose glide component is the external shear stress resolved on the slip plane in the direction of the Burgers vector multiplied by the magnitude of the Burgers vector, b. This can be called the external force.

There will also be a force arising from the rate of change of the interaction energy between the stress field of the dislocation itself and the stress fields of all other sources of internal stress. This is also equal to the local value of the shear stress arising from the sources of internal stress resolved on the slip plane and in the direction of the Burgers vector multiplied by b, and is the force due to the internal stress.

In addition there are forces which arises from the rate of change of the energy stored in the stress field of the dislocation itself as it moves. The line tension force arises from the changes in the infinite body part of the dislocation stress field. On the simplest model the force is equal to Γ/r where Γ is the energy stored in the stress field per unit length of dislocation and r is the radius of curvature of the dislocation line. Changes in the energy stored in that part of the stress field of the dislocation which arises from satisfying the boundary conditions in a finite body, result in image forces. Image forces may not be without significance in the creep of polycrystalline or multiphase materials but they will be ignored here. Line tension and image forces cannot be calculated by multiplying an actual component of stress by $\boldsymbol{b}$, but it is possible to calculate a fictitious equivalent stress by dividing the force by $\boldsymbol{b}$.

Last of all, there are forces due to non-linear elastic effects in the dislocation core causing the core energy to vary as the dislocation changes position with respect to the crystal lattice. These are lattice friction forces. They are periodic and the maximum value must be reached before movement of more than a few Burgers vectors will take place. Friction forces always oppose dislocation motion if this is defined to be a displacement of more than a few Burgers vectors and the friction force is then the maximum value. The forces due to a distribution of point defects behave in a similar manner and it is convenient to lump these two sources of force under the heading of friction forces.

The total force, f, on a dislocation, if image forces are ignored, is therefore

$$f = f_a + f_i + f_T + f_F \qquad (2)$$

where f_a, f_i, f_T and f_F are the forces due to the external stress, the internal stress, line tension and friction, respectively. The dislocation will be in equilibrium when f is zero. In general, the equilibrium will be stable but some dislocation configurations are at or near an unstable equilibrium condition and a small additional displacement will cause $|f|$ to increase. The dislocation will then accelerate and move relatively large distances. A dislocation bent into a semi-circular loop is an instance of such an unstable configuration. A small displacement away from the centre of curvature will cause $f_T(\approx T/r)$ to decrease while the other terms in Equation 2 remain constant. The loop then expands freely in the Frank–Read manner.

If $-\tau_i$ is the local internal shear stress, τ_a the externally applied stress (both resolved on the slip plane and in the direction of the Burgers vector) and τ_F a friction stress given by $-f_F/b$, then Equation 2 may be written

$$f = (\tau_a - \tau_i - \tau_F)b + f_T \qquad (3)$$

where the sign convention used in Equation 1 has been employed for both τ_i and τ_F.

For any given dislocation configuration there will be a value of f_T which is the unstable equilibrium value. If this value is divided by b, then a local threshold stress, $-\tau_{th}$, is obtained and the condition for local plastic flow is

$$(\tau_a - \tau_i - \tau_F) \geq \tau_{th} \qquad (4)$$

Macroscopic deformation occurs when sufficient local regions have undergone plastic flow to enable this to be detected by the strain measuring apparatus. As already noted, if a value of internal stress is to survive the passage from the local region to the whole body then the averaging process must be weighted in some way. There are two main ways in which dislocations give weighted averages of the internal stress.

(a) Temporal weighting: dislocations spend less time in regions where the internal stress assists the applied stress (i.e. regions of high effective stress) than they do in regions where the effective stress is low. The average internal stress experienced by the dislocations is thus non-zero and is such as to oppose the applied stress.

(b) Spatial weighting: in this kind of weighting there are regions of high effective stress ("hard" regions) in which the dislocations cannot move as easily as in other regions where the effective stress is low ("soft" regions). The amount of dislocation movement is thus greater in regions of low effective stress which are given greater weight, and a non-zero average internal stress results. Weighted average internal stresses of this sort give rise to phenomena such as the Bauschinger effect.

Hence it is expected that there are average (tensile) values of σ_i, $\sigma_e(= \sigma_a - \sigma_i)$ and $\sigma_a - \sigma_i - \sigma_F$ at the macroscopic level which are, in principle, detectable in macroscopic experiments. In the same way there is an average value for the (tensile) threshold stress, σ_{th}. It will be seen later that other factors are also involved in the weighting of the macroscopic averages.

This serves to define the quantities, σ_a, σ_i, σ_F and σ_{th}. Other terms are also used to designate various combinations of the quantities defined here. In particular the *back stress*, σ_b, is defined by

$$\tau_b = \tau_i + f_T/b \qquad (5)$$

where we have now adopted the same sign convention for f_T/b as for internal stress. The corresponding macroscopic average (tensile) value, σ_b, of the back stress is also a useful quantity. The term internal stress is often used for the quantity here called back stress and in many cases there is ambiguity as to which reference is being made.

The total force on a dislocation (excluding image forces) is $\tau_g b$ where τ_g (to be called the glide stress) can be regarded as a generalization of the effective stress. It is given by

$$\tau_g = \tau_a - \tau_b = \tau_a - \tau_i - f_T/b \qquad (6)$$

and it may be expected that there are corresponding macroscopic quantities, σ_g and σ_b, such that $\sigma_g = \sigma_a - \sigma_b$.

3. Recovery-controlled and glide-controlled creep

Recovery is the result of thermally activated processes which change the values of σ_i and σ_{th} which determine the flow stress. The average velocity of dislocation glide may be controlled by such processes. When this is the case creep is said to be recovery controlled.

There are two classes of model for recovery creep. In the first of these bursts of rapid glide punctuate long periods of relatively small dislocation movement during which recovery occurs. This is the so-called jerky or athermal glide model. During the recovery period the dislocation is in the stable state defined by Equation 2 with $f = 0$, but recovery will eventually bring it to the unstable state defined by Equation 4. If the friction stress, σ_F, is such that the energy barrier it presents to dislocation motion is small relative to the thermal energy available, then, once the unstable state has been achieved, the dislocation will move in an athermal manner, accelerating to high speed and moving a relatively large distance until it meets an obstacle which arrests it. Recovery will then change the values of τ_i and f_T and the segment may be released for a further glide event. Clearly the time taken for the thermally activated processes will be large compared to the essentially athermal glide events and the overall strain rate will be determined by the rate of recovery. If the friction force is not negligibly small but can still be overcome by thermal activation, then dislocation motion will no longer be athermal. Creep will still be recovery controlled if the time taken for the dislocation to glide after it has been released is short relative to the time spent waiting to be released by recovery. As the friction stresses become more significant there will be a transition from recovery control to glide control of the creep rate.

In the second class of models for recovery creep, jerky or athermal glide is not involved and dislocations are always at or near equilibrium. This could be the case if, for example, the sub-grain size or the grain size is smaller than the length of dislocation which would be unstable at the given local effective stress. Recovery may progressively release a segment from one sub-grain wall and allow it to expand slowly across the sub-grain until it meets and is possibly incorporated into the opposite sub-grain wall without ever undergoing athermal glide. This is the continuous glide recovery model.

A different case arises if the energy barriers due to friction are so large that glide cannot be thermally activated at a rate that can be detected. In this case the friction stress, as it has been defined, will be so large that it is not possible for the equilibrium expressed by putting $f = 0$ in Equations 2 and 3 to be satisfied. Equilibrium will, of course, be achieved by small movements within the limit of a few Burgers vectors used to define the friction stress, but greater movements than this will not be possible and there will be no creep due to dislocation glide. However, there is an alternative mechanism for dislocation movement if the major part of the friction stress arises from interaction with point defects. Diffusion of the point defects in response to the force exerted on them by the dislocation enables slow glide to take place and creep will again be glide controlled.

There are other cases in which creep may be glide controlled which cannot be described in terms of a friction stress. When the material has a fine dispersion of particles which cannot be cut or looped, dislocations can only glide if short segments climb or cross-slip over the particles and the rate of these processes will determine the average rate of glide. In other cases, dislocations may only be able to move if point defects are created at jogs and moved away into the lattice with the help of thermal activation.

4. Anelastic effects

Anelasticity is defined as time-dependent recoverable strain. Unloading experiments will therefore always involve anelasticity. The view taken in the present paper is that anelasticity is a manifestation of plasticity and is not distinct from it, although it is often found convenient to make such a distinction from a phenomenological point of view. Anelasticity arises because some plastic deformation mechanisms store elastic energy in the deformed body which can be used to do work against the external forces and cause reverse strain on partial unloading. Anelasticity is therefore an aspect of the general phenomenon being discussed here. Some workers have suggested that the zero strain rate condition determined by the strain transient dip test is simply the point when the reverse strain due to anelasticity balances the continuing forward strain due to normal creep [21, 22]. This view seems to imply that there is a difference between plasticity and anelasticity, a view which the present authors do not hold. However, because of the heterogeneous nature of the deformation process and the resulting variation of internal stress from point to point, it is to be expected that a large stress reduction will cause some regions to strain in the reverse direction while others continue to strain in the forward sense. Hence the statement that a zero creep rate after a stress reduction is a result of superposition of forward and backwards creep is one that cannot be argued with. It is to be expected that this process of superposition defines the average internal stress.

Difficulties of interpretation arise, however, when deformation mechanisms which do not contribute significant amounts of strain in normal creep, or which are not themselves rate-controlling, contribute a significant amount of reverse creep on unloading. Grain-boundary sliding in polycrystals is a case in point. This relaxes the shear stress on grain boundaries to very low values during normal creep, generating internal stresses in the grains which are in the same sense as the applied stress. The grain boundaries thus play the role of the soft component of a composite system, with very low effective stresses, and the grains the hard component. Partial unloading will bring about a negative effective shear stress on the boundaries which therefore slide in the direction giving reverse strain. In this process some of the strain energy stored in the grains will be used to raise the potential energy of the loading system and some will be degraded into heat. It will be noted that this is exactly analogous to reverse creep by the main or rate-controlling mechanism occurring after unloading. Forward creep stores strain energy in internal stress fields which then drives reverse creep after unloading.

If all possible deformation mechanisms could be taken into account in the description of forward creep, a complete description of unloading transients would be possible without invoking a third deformation mode different from elasticity or plasticity. At present this can only be done to a very limited degree. In the following section the effect of various mechanisms which can give reverse strain in addition to the main deformation mode will be briefly discussed.

5. Interpretation of the dip test

5.1. Glide-controlled case

The force per unit length exerted by a dislocation on the obstacles giving rise to the friction stress is $\tau_g b$ where

$$\tau_g = \tau_a - \tau_i - f_T/b$$
$$= \tau_a - \tau_b \quad (7)$$

where τ_a and τ_i are the resolved components of the applied stress and the internal stress, respectively, and τ_b is the back stress. In terms of the applied tensile stress, σ_a

$$\tau_a = \sigma_a \cos\theta \cos\psi$$
$$= \sigma_a m \quad (8)$$

where θ and ψ are defined in Fig. 1 and $m = \cos\theta\cos\psi$ is the orientation factor.

If τ_g is averaged along the whole length of dislocation line in the material the value obtained is

$$\langle\tau_g\rangle = \langle m\rangle\sigma_a - \langle\tau_b\rangle \quad (9)$$

and σ_g and σ_b could be defined as $\langle\tau_g\rangle/\langle m\rangle$ and $\langle\tau_b\rangle/\langle m\rangle$, respectively. However, as will be seen, these are *not* the averages defined by the dip test.

The length, dl, of dislocation line having τ_g between τ_g and $\tau_g + d\tau_g$ and m between m and $m + dm$ is given by

$$dl = \rho V p(\tau_g, m)\, d\tau_g\, dm \quad (10)$$

where p is a normalized probability distribution, V the total volume and ρ the *total* dislocation density. It will become clear that the whole of the dislocation population has to be included in the averaging process if the dip test is to be correctly analysed. If the velocity of an element of dislocation line is

$$v = v(\tau_g) \quad (11)$$

the tensile strain rate is

$$\dot{\varepsilon} = \rho b \int\int v(\tau_g) m p\, d\tau_g\, dm \quad (12)$$

The integral part of Equation 12 is the orientation weighted average of the velocity, $\tilde{v} = \langle v(\tau_g)m\rangle$. Note that many dislocations with zero velocity are included in the average, $\tilde{v}$. Because v and m are correlated, $\tilde{v} \neq \langle v\rangle\langle m\rangle$. We have

$$\dot{\varepsilon} = \rho b \tilde{v} \quad (13)$$

The value of $\tilde{v}$ defines an average value of τ_g ($= \tilde{\tau}_g$) via the inverse functional relationship of Equation 11, i.e.

$$\tilde{\tau}_g = \tau_g(\tilde{v}) \quad (14)$$

In general, there is insufficient information in Equations 12 and 14 to allow $\tilde{\tau}_g$ to be partitioned into components due to the applied stress and due to the back stress. However, if v is proportional to τ_g, i.e. $v = k\tau_g$ we have

$$\tilde{v} = \int\int k(m\sigma_a - \tau_b) m p\, d\tau_g\, dm \quad (15)$$

or

$$\tilde{v}/k = \tilde{\tau}_g = \langle m^2\rangle\sigma_a - \langle\tau_b m\rangle \quad (16)$$

which may be compared with Equation 9. The definitions

$$\sigma_g = \tilde{\tau}_g/\langle m^2\rangle \quad (17)$$

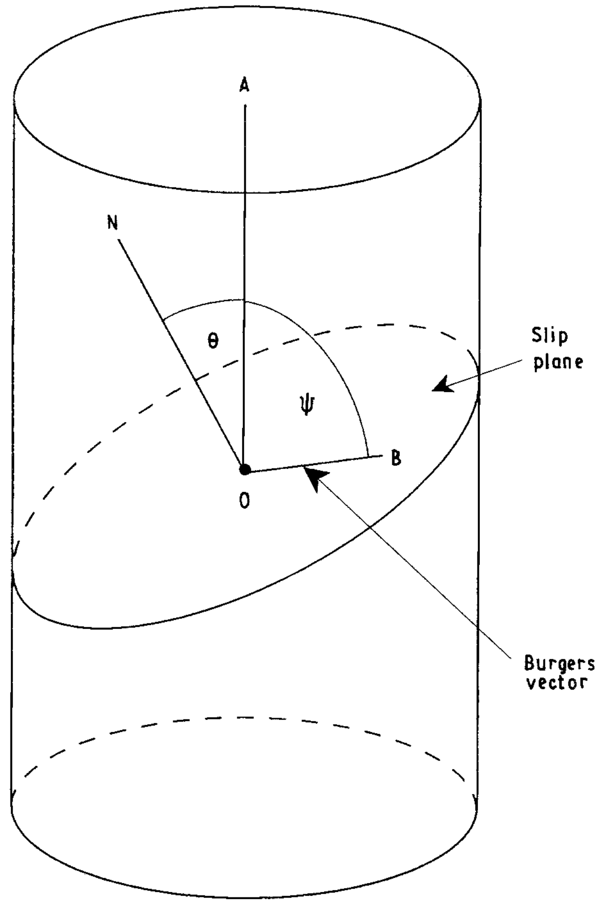

Figure 1 The angles, θ and ψ, defining the orientation factor, m, for a dislocation. OA is the tensile axis, ON the normal to the slip plane and OB the Burgers vector. OA, ON and OB are not co-planar in general.

and

$$\sigma_b = \langle\tau_b m\rangle/\langle m^2\rangle \quad (18)$$

then give

$$\sigma_g = \sigma_a - \sigma_b \quad (19)$$

Suppose now that the applied stress is reduced by $\Delta\sigma_a$. The new strain rate immediately after the change is

$$\dot{\varepsilon}' = \rho b \int\int v(\tau_g - m\Delta\sigma_a) m p\, d\tau_g\, dm \quad (20)$$

where p and τ_g are the values obtaining immediately before the stress reduction. This defines a new average $\tau_g (= \tilde{\tau}'_g)$ given, in the linear case, by

$$\tilde{\tau}'_g = \langle m^2\rangle(\sigma_a - \Delta\sigma_a - \sigma_b) \quad (21)$$

The new stain rate will be zero when $\tilde{\tau}'_g$ is zero, i.e. when

$$\sigma_a - \Delta\sigma_a = \sigma_b \quad (22)$$

and

$$\Delta\sigma_a = \sigma_g \quad (23)$$

Clearly the remaining stress when $\dot{\varepsilon}' = 0$ is equal, not to the average internal stress, but to the average *back stress*, σ_b. The experiment does not allow σ_b to be split up into components due to internal stress and line tension.

In the non-linear case it is impossible to write equations equivalent to Equations 21–23. When $\dot{\varepsilon}' = 0$ the stress reduction, $\Delta\sigma_a$, will have made $\tilde{\tau}'_g$ equal to zero but the remaining stress, $\sigma_a - \Delta\sigma_a$, cannot be interpreted as the mean tensile back stress before stress reduction nor can the stress reduction, $\Delta\sigma_a$, be interpreted as the mean tensile glide stress. For example, if v were proportional to τ_g^3, Equations 12 and 14 would give

$$\tilde{\tau}_g = [\langle m^4 \rangle \sigma_a^3 - 3\langle m^3 \tau_b \rangle \sigma_a^2 + 3\langle m^2 \tau_b^2 \rangle \sigma_a - \langle m \tau_b^3 \rangle]^{1/3} \qquad (24)$$

from which it is clear that no separation of $\tilde{\tau}_g$ into applied stress and back stress terms is possible. This results from the fact that the average value, $\tilde{\tau}_g$, is determined not only by the distribution of m and τ_g but also by the functional relationship between τ_g and velocity, v.

Because the total, rather than the mobile, dislocation density is used in Equations 12 and 13, the average velocity defined in Equation 12 will be much smaller than the average velocity of the mobile dislocations alone. Similarly, the average τ_g will be smaller than the value for the mobile dislocations. This procedure is important because, although the non-mobile part of the dislocation population makes no contribution to the strain before unloading, after unloading these dislocations will straighten at a velocity governed by Equation 11, causing reverse strain in the process. It is essential that this should be included in the analysis if the significance of the results of the dip test is to be understood. Reverse strain arising from previously immobile dislocations would normally be called anelastic strain and this particularly simple case is therefore an example of how anelastic strain can be treated in the frame work of a complete theory of forward creep. It has to be so treated if the results of the dip test are to be correctly interpreted. Clearly the dip test determines the value of σ_b for *all* the dislocations, mobile and immobile.

The individual contributions of the mobile and immobile dislocations of σ_b can be obtained as follows. From Equation 10 it can be shown that

$$\rho\langle \tau_b m \rangle = \rho_I \langle \tau_b m \rangle_I + \rho_M \langle \tau_b m \rangle_M \qquad (25)$$

where the subscripts I and M refer to the immobile and mobile populations, respectively. For the immobile population, $\tau_g = 0$ by definition, which gives $\tau_b = m\sigma_a$ and hence

$$\rho\langle \tau_b m \rangle = \rho_I \langle m^2 \rangle_I \sigma_a + \rho_M \langle \tau_b m \rangle_M \qquad (26)$$

or

$$\sigma_b = \frac{\langle \tau_b m \rangle}{\langle m^2 \rangle} = \sigma_a - \Delta\sigma_a$$

$$= \frac{\rho_I \langle m^2 \rangle_I \sigma_a}{\rho \langle m^2 \rangle} + \frac{\rho_M \langle \tau_b m \rangle_M}{\rho \langle m^2 \rangle} \qquad (27)$$

This treatment of anelastic effects is strictly only valid if the spacing of the sources of the friction stress along the dislocation is very much smaller than its radius of curvature, such as would be the case for a solute atom atmosphere. However if the motion of the dislocation is controlled by dragging defect emitting/absorbing jogs then it will be curved between the jogs. Changes in this curvature will cause reverse strain on unloading which will not be taken into account by the above treatment, although such changes in curvature are likely to be completed during unloading and therefore before the start of reverse creep in most cases.

The treatment also applies strictly only to single crystals. In polycrystals some reverse strain on unloading is caused by grain-boundary sliding and this is not taken into account. However, in the Appendix it is shown that reverse grain-boundary sliding occurs so rapidly in many cases that it is virtually complete before the first readings of strain are taken after a stress reduction. Hence its effect on the values of σ_g and σ_b obtained from the strain transient dip test can often be neglected.

5.2. Recovery-controlled cases

It is assumed that dislocations are in a three-dimensional network with a distribution of segment or link lengths, L. The resolved effective stress on a link is $\tau_e = \tau_a - \tau_i$ as before. Each link is characterized by a parameter, x, where

$$x = \frac{\tau_e}{\tau_{th}} = \frac{L}{2R} \qquad (28)$$

where

$$\tau_{th} = 2\alpha G b / L \qquad (29)$$

is the Frank–Read threshold stress with α a constant $\approx 1/2$, G the shear modulus and b the Burgers vector, while R is the equilibrium radius of curvature given by

$$R = \alpha G b / \tau_e \qquad (30)$$

The threshold stress, τ_{th}, is regarded as an intrinsically positive quantity so that $x = 1$ is the condition for instability in the forward direction and $x = -1$ for instability in the reverse direction.

Recovery changes τ_{th} (via changes in L) and τ_e (via changes in τ_i), allowing strain to take place. There is a local recovery rate, $\dot{x} = dx/dt$, for each segment.

As already explained there are two ways in which strain may occur in recovery-controlled creep, continuous glide and jerky glide. In the former case the dislocation links sweep out additional area as recovery changes both τ_e (via changes in τ_i) and τ_{th} (via changes in L). The processes are illustrated in Fig. 2. For continuous glide the strain rate is determined by $\dot{A} = dA/dt$, the area swept out per unit time by a dislocation link. We have

$$\dot{A} = \frac{\partial A}{\partial \tau_e}\dot{\tau}_e + \frac{\partial A}{\partial \tau_{th}}\dot{\tau}_{th} \qquad (31)$$

and $\dot{A}$ is therefore a function of τ_e, $\dot{\tau}_e$, τ_{th} and $\dot{\tau}_{th}$. It is not necessary to use the actual functional relationship for the present purposes.

The tensile strain rate is therefore

$$\dot{\varepsilon} = bN \int\int\int\int\int m\dot{A}q_1 \, dm \, d\tau_e \, d\dot{\tau}_e \, d\tau_{th} \, d\dot{\tau}_{th} \qquad (32)$$

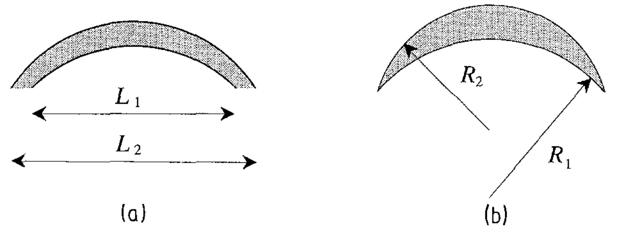

Figure 2 Illustrating the strain-producing processes in continuous glide recovery creep. (a) A dislocation segment initially of length L_1 is bowed into an arc of radius R by the effective stress. The shaded area is swept out if recovery changes L_1 to L_2 (i.e. changes τ_{th}) while the internal stress, and hence the effective stress and R, remain constant. (b) A dislocation segment, initially bent into a radius R_1, sweeps out the shaded area as recovery reduces the internal stress, increasing the effective stress and changing the radius to R_2 while the segment length, L, remains constant. In general both these processes will act on any given segment simultaneously.

where q_1 is the probability distribution of m, τ_e, $\dot{\tau}_e$, τ_{th} and $\dot{\tau}_{th}$ for the links. The integral is the orientation weighted average of $\dot{A}$, $\langle m\dot{A}\rangle$.

However, $\langle m\dot{A}\rangle$ does not define a unique mean value of effective stress, τ_e, as the equivalent quantity, $\langle mv\rangle$, defines $\bar{\tau}_g$ in the glide-controlled case. This is because $\dot{\tau}_e$ and $\dot{\tau}_{th}$ do not depend simply on the effective stress on the link in question. For instance, $\dot{\tau}_{th}$ will depend partly on the effective *tensile* stresses on a large number of links surrounding the particular link, as these climb to cause network coarsening, increasing L. Similarly, $\dot{\tau}_i$ (and hence $\dot{\tau}_e$) will be determined by changes, driven in part by an effective stress, in the arrangements of geometrically necessary dislocations quite remote from the link in question.

If the stress is reduced, dislocations will increase their radii of curvature and straighten. For large reductions the effective stress on some links will change sign and continued network coarsening will allow these links to contribute reverse strain. It is conceivable that there will be a stress reduction, $\Delta\sigma_a$ which will make $\langle m\dot{A}\rangle$, and therefore $\dot{\varepsilon}$, temporarily zero. Clearly there will be no simple relationship between the stress remaining, $\sigma_a - \Delta\sigma_a$ and the average internal stress experienced by the links.

In the jerky glide model, recovery brings links to the unstable state $x = 1$ and bursts of athermal glide occur during which the released dislocation sweeps out an area A_j producing strain. In general, both this and continuous glide will occur simultaneously but only the case where jerky glide contributes most of the total strain will be considered. If the number, dn, of links per unit volume having x from x to $x + dx$, m from m to $m + dm$, $\dot{x}$ from $\dot{x}$ to $\dot{x} + d\dot{x}$ and A_j from A_j to $A_j + dA_j$ is

$$dn = N q_2(x, m, \dot{x}, A_j)\, dx\, dm\, d\dot{x}\, dA_j \quad (33)$$

where N is the total number of links per volume, then the strain rate is given by

$$\dot{\varepsilon} = bN \int\int\int mA_j\dot{x}_{x=1}\, q_2(x = 1, m, \dot{x}, A)\, dm\, d\dot{x}\, dA \quad (34)$$

because only links with $x = 1$ make a contribution to the strain rate. The integral is the average value of $mA_j\dot{x}_{x=1}$ so that

$$\dot{\varepsilon} = bN\langle mA_j\dot{x}_{x=1}\rangle \quad (35)$$

As in the previous case, A_j and $\dot{x}$ depend upon the effective stress in different ways. A_j depends on the local effective stress on the link (and also on the arrangement of links in the local region) but because $\dot{x}$ is related to $\dot{\tau}_e$ and $\dot{\tau}_{th}$ it will depend on the effective stresses in a relatively large and ill-defined volume as already discussed. Hence the mean value $\langle mA\dot{x}_{x=1}\rangle$ will not define a unique value of the mean effective stress as was the case for the corresponding quantity in the glide-controlled model.

If the stress is reduced, jerky glide will immediately cease because x will have been reduced for all links and there will be no links at the unstable condition, $x = 1$. There will be reverse strain as dislocations straighten but this will be very rapid if glide is truly athermal and will occur as rapidly as the load can be reduced. There will also be time-dependent reverse strain due to grain-boundary sliding as considered in the Appendix. Continuous glide recovery creep will still give forward creep for small stress reductions and possibly negative creep for large stress reductions. The effect of the stress reduction, as far as the jerky glide mechanism is concerned, is to shift the distribution function q_2 along the x axis in a negative direction and also change its shape. For a large enough stress reduction the left-hand tail of the distribution will overlap the point $x = -1$ and rapid reverse strain will occur followed by a period of reverse creep by the jerky glide mechanism. Rapid reverse yield followed by reverse creep was observed after stress reductions during the creep of aluminium in both single and polycrystalline form in a series of very careful experiments by Parker and Wilshire [23]. Their results are illustrated schematically in Fig. 3.

The analysis of the strain transient dip test for the glide-controlled case is based on averaging the dislocation velocity. When jerky glide recovery is applicable it is possible to analyse the reverse yield stress by averaging the condition for Frank–Read instability for the links contributing the strain. During forward creep we can average the effective stress, $\tau_e = m\sigma_a - \tau_i$

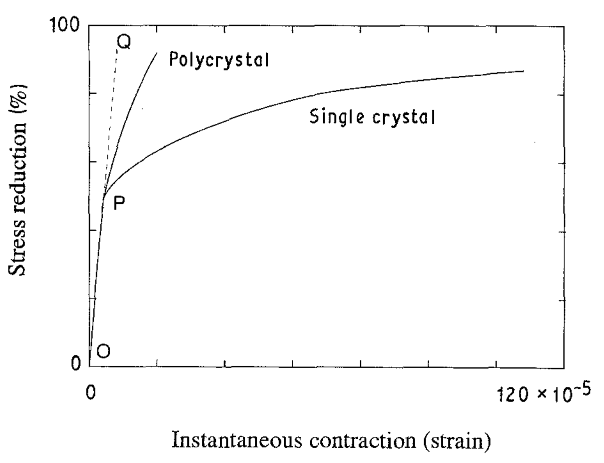

Figure 3 Schematic representation of reverse yield on unloading super-purity aluminium creeping at 573 K with an initial stress of 10.3 MN m^{-2} (after Parker and Wilshire [23] Fig. 6). OQ is the elastic unloading line and the remaining stress corresponding to the point P is the reverse yield stress.

over the links with $x = 1$ to obtain

$$\langle\tau_e\rangle = \langle\tau_{th}\rangle = \langle m\rangle\sigma_a - \langle\tau_i\rangle \quad (36)$$

or, defining $\sigma_{th} = \langle\tau_{th}\rangle/\langle m\rangle$ and $\sigma_i = \langle\tau_i\rangle/\langle m\rangle$

$$\sigma_{th} = \sigma_a - \sigma_i \quad (37)$$

If the stress reduction, $\Delta\sigma_a$, is just sufficient to give reverse creep with negligible reverse yield (i.e. $\sigma_a - \Delta\sigma_a$ is the reverse yield stress) then the effective stress, σ_e, will be negative and equal to $-\sigma'_{th}$ where σ'_{th} (which is intrinsically positive) is the average value for the dislocations involved in reverse creep. We can write for these dislocations

$$-\sigma'_{th} = \sigma_a - \Delta\sigma_a - \sigma'_i \quad (38)$$

From Equations 37 and 38 we have

$$\sigma_{th} - \sigma'_{th} = 2\sigma_a - \Delta\sigma_a - (\sigma_i + \sigma'_i) \quad (39)$$

and

$$\sigma_{th} + \sigma'_{th} = \Delta\sigma_a + (\sigma'_i - \sigma_i) \quad (40)$$

Equations 39 and 40 are the equivalent of Equations 22 and 23 for the glide-controlled case. However, they are far less informative because there are four unknown quantities and only two equations connecting them. It is clear that $\sigma_i \neq \sigma'_i$ since links with a low internal stress are likely to be involved in forward creep while those causing reverse creep will have a high internal stress. However, for both forward and reverse creep, it is links with the lowest value of τ_{th} which will give strain. It is possible therefore that $\sigma_{th} = \sigma'_{th}$. If this is the case then Equations 39 and 40 become

$$\sigma_a - \tfrac{1}{2}\Delta\sigma_a = \tfrac{1}{2}(\sigma'_i + \sigma_i) \quad (41)$$

and

$$\sigma_{th} = \tfrac{1}{2}\Delta\sigma_a + \tfrac{1}{2}(\sigma'_i - \sigma_i) \quad (42)$$

and the reverse yield stress allows an average value of the internal stresses experienced by the links involved in both forward and reverse creep to be calculated.

If the stress reduction, $\sigma_a - \Delta\sigma_a$, is greater than that required to reach the reverse yield stress some rapid reverse strain will take place and reverse creep will follow. Equation 38 will be valid for the dislocations involved in this reverse creep but the values of σ'_{th} and σ'_i will be different from those applicable to the material for which $\sigma_a - \Delta\sigma_a$ is the reverse yield stress. This is because the rapid reverse strain preceding the reverse creep will change the structure and hence change both σ'_{th} and σ'_i. It will then certainly not be possible to put $\sigma_{th} = \sigma'_{th}$ even if this were true for the reduction $\Delta\sigma_a$ which just brings the stress to the reverse yield stress.

7. Discussion and conclusions

For glide-controlled creep it has been shown that the strain transient dip test can only be interpreted if the relation between dislocation velocity and the glide stress is linear. When this is the case the expression for the creep rate defines a unique value for the average *back stress* exerted on all the dislocations, mobile or immobile, and the strain transient dip test enables this average back stress to be determined. The back stress is the resolved component of the internal stress plus the line tension glide force divided by the Burgers vector, but the dip test does not give enough information to allow the separation of the back stress into components due to the internal stress and due to line tension. Hence it has been shown that the strain transient dip test determines an average back stress *not* an average internal stress as usually assumed, and this only if the dislocation velocity is proportional to the glide stress.

When dislocation velocity is non-linearly related to the glide stress, a stress reduction can be found which gives zero creep rate and this stress reduction does make the average glide stress equal to zero. However, it has been shown that the remaining stress, $\sigma_a - \Delta\sigma_a$, is *not* equal to the average (tensile) back stress and the stress reduction, $\Delta\sigma_a$, is not equal to the average (tensile) glide stress before stress reduction.

For recovery-controlled creep the expressions for the creep rate do not define a unique average value for the internal stress or the back stress. This is true for both continuous glide and jerky glide recovery models. It follows that the strain transient dip test cannot be used to determine an average internal stress or average back stress for materials undergoing recovery-controlled creep. For recovery creep involving continuous glide, a series of dip tests may enable a stress reduction to be found which gives a zero creep rate, but there is no simple interpretation of the stress remaining in terms of an average back stress or average internal stress. Nevertheless the observation of negative creep in such materials would indicate the importance of internal stress in determining creep behaviour.

For jerky glide recovery creep the strain transient dip test fails because creep (at least by the principle rate-controlling mechanism) ceases as soon as the stress is reduced by any amount. If internal stresses are sufficiently large there will be a stress reduction large enough to cause rapid reverse glide followed by reverse creep, i.e. there is a reverse yield stress. It was pointed out by Poirier [24] some time ago that the occurrence of reverse yield on unloading was better evidence for the jerky glide recovery model than the existence of incubation periods, which are the subject of continuing controversy [25–27].

The determination of the reverse yield stress for a material undergoing jerky glide recovery creep is the equivalent experiment to the strain transient dip test for a material undergoing glide-controlled creep. The results of both of these kinds of experiment show that internal stresses are important in creep. Only a limited amount of information can be obtained from measurements of the reverse yield stress, however. The limitation is related to the fact that reverse creep involves a different set of dislocations experiencing different internal stresses from those involved in forward creep. If the threshold stresses for the two sets of dislocations are the same, which seems plausible, then it is possible to calculate the average internal stress on the dislocations involved in both forward and reverse creep, but average internal stresses for the two sets of dislocations separately cannot be obtained.

Appendix

Here we calculate the reverse strain and the reverse strain rate due to grain-boundary sliding when a polycrystalline specimen is partially unloaded during creep. On first loading a polycrystal at high temperatures, grain boundaries, slide very rapidly, relaxing the shear stresses on them to very low values. The displacements, and hence the strains they cause, are very small and the sliding rate is a consequence of the intrinsic properties of the boundary. Thereafter sliding can only take place at a rate which is controlled by accommodating creep deformation in the grains and is very much slower. When the specimen is unloaded by $\Delta\sigma_a$ the shear stress on a boundary changes from zero to $-m\Delta\sigma_a$ and reverse sliding takes place to reduce it to zero again. This sliding takes place at the very high rate determined by the intrinsic "viscosity" of the boundary and purely elastic accommodation of the grains. To calculate the resulting reverse strain rate and maximum reverse strain we use the formal theory outlined by Nowick and Berry [28].

If u is the relative tangential displacement of a pair of grains then

$$\dot{u} = -(u - u_m)/t_r \qquad \text{(A1)}$$

where $\dot{u} = du/dt$, u_m, the maximum displacement determined by elastic accommodation and t_r is a characteristic time (the relaxation time) given by

$$t_r = t_o \exp(\Delta h/RT) \qquad \text{(A2)}$$

where t_o is a constant, Δh is a molar activation enthalpy, R is the gas constant ($= 8.314\ \mathrm{J\,mol^{-1}\,K^{-1}}$) and T is thermodynamic temperature . From Equation A1

$$u = u_m[1 - \exp(t/t_r)] \qquad \text{(A3)}$$

and

$$\dot{u} = (1/t_r)u_m[1 - \exp(-t/t_r)] \qquad \text{(A4)}$$

We also have

$$\begin{aligned} u_m &= \tau d/G \\ &= m\Delta\sigma_a d/G \qquad \text{(A5)} \end{aligned}$$

where d is a linear grain dimension (the grain size) and G the shear modulus.

Consider a facet of grain boundary, which we assume to be plane. The strain from this facet is

$$\delta\varepsilon = aum/V \qquad \text{(A6)}$$

If there are n facets in the volume, V, then using Equation A2, the total strain is

$$\varepsilon = \frac{n}{V}\langle adm^2\rangle \frac{\Delta\sigma_a}{G}[1 - \exp(-t/t_r)] \qquad \text{(A7)}$$

When $t = \infty$, $\varepsilon = \varepsilon_m$, the maximum value, i.e.

$$\varepsilon_m = \frac{n}{V}\langle adm^2\rangle \frac{\Delta\sigma_a}{G} \qquad \text{(A8)}$$

The average value could be calculated for a specific model of grain shape, but it is more convenient to use Zener's result [29] for the ratio of the relaxed and unrelaxed tensile moduli (E_R and E, respectively). This is

$$\frac{E_R}{E} = \frac{(7 + 5\nu)}{2(7 + \nu - 5\nu^2)} \qquad \text{(A9)}$$

where ν is Poisson's ratio. Because

$$\varepsilon_m = \Delta\sigma_a\left(\frac{1}{E_R} - \frac{1}{E}\right) \qquad \text{(A10)}$$

we have

$$\begin{aligned} \varepsilon_m &= \frac{\Delta\sigma_a}{E}\left(\frac{7 - 3\nu - 10\nu^2}{7 + 5\nu}\right) \\ &= \frac{\Delta\sigma_a k}{E} \qquad \text{(A11)} \end{aligned}$$

where $k \approx 0.56$ if $\nu = 1/3$. Using Equations A11 and A8 in Equation A7 we obtain

$$\varepsilon = \frac{\Delta\sigma_a k}{E}[1 - \exp(-t/t_r)] \qquad \text{(A12)}$$

and

$$\dot{\varepsilon} = \frac{\Delta\sigma_a k}{E}\frac{\exp(-t/t_r)}{t_r} \qquad \text{(A13)}$$

The time, $t_{0.99}$, for the strain to reach 99% of the final value, ε_m, is, from Equations A12 and A11

$$t_{0.99} = t_r \ln 100 \qquad \text{(A14)}$$

and the reverse strain rate, $\dot{\varepsilon}_{0.99}$, at $t_{0.99}$

$$\dot{\varepsilon}_{0.99} = \frac{\Delta\sigma_a k}{100 E t_r} \qquad \text{(A15)}$$

From Equation A13 the initial strain rate, $\dot{\varepsilon}_i$, immediately after unloading is

$$\dot{\varepsilon}_i = \frac{\Delta\sigma_a k}{E t_r} \qquad \text{(A16)}$$

These results can be used to carry out some illustrative calculations. Williams and Leak [30] give for

TABLE I

T(K)	T/T_m	t_r (s)	E (GPa)	$\dot{\varepsilon}_i$ (s^{-1})	$\dot{\varepsilon}_{0.99}$ (s^{-1})	$t_{0.99}$ (s)
425	0.46	384.1	72	1.0×10^{-7}	1.0×10^{-9}	1769.0
475	0.51	5.43	70	7.4×10^{-6}	7.8×10^{-8}	25.0
525	0.56	0.17	66	2.5×10^{-4}	2.4×10^{-6}	0.8
575	0.62	0.098	59	4.75×10^{-3}	4.75×10^{-5}	0.06

99.999% aluminium, $t_o = 10^{-15}$ s, and $\Delta h = 143$ kJ mol^{-1}. Table I gives values of $t_{0.99}$, $\dot{\varepsilon}_{0.99}$ and $\dot{\varepsilon}_i$ for a stress reduction of 5 MPa and various temperatures using these data and values of Young's modulus from Beaton and Hewitt [31].

It is clear from Table I that, for tests above about 500 K in very high-purity aluminium, grain-boundary relaxation is completed so rapidly that its effects will not be noticed in a strain transient dip test. Unloading must take at least 1 s if dynamic effects are to be avoided and the first readings of strain are likely to be several seconds after the start of the unloading process. It should be noted that although $\dot{\varepsilon}_i$ and $\dot{\varepsilon}_{0.99}$ depend on $\Delta\sigma_a$, $t_{0.99}$ does not.

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